Julia-Toulouse approach to $(d+1)$-dimensional bosonized Schwinger model with an application to large N QCD

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The Julia-Toulouse approach for condensation of charges and defects is used to show that the bosonized Schwinger model can be obtained through a condensation of electric charges in $1+1$ dimensions. The massive model is derived by taking into account the presence of vortices over the electric condensate, while the massless model is obtained when these vortices are absent. This construction is then straightforwardly generalized for arbitrary $d+1$ spacetime dimensions. The $d=3$ case corresponds to the large N chiral dynamics of $SU(N)$ QCD in the limit $N\to\infty$.

Keywords: Schwinger model, bosonization, condensate, vortices, instantons, large N approximation.

I. INTRODUCTION

Schwinger model [1, 2] is the name given to electrodynamics in $1+1$ dimensions. This model was originally examined by Schwinger as an example where mass and gauge invariance coexists compatibly. While the classical theory is confining due to the linear behavior of the Coulomb interaction in $1+1$ dimensions, quantum effects can modify this picture. The quantum theory with massless fermions is exactly solvable (i.e., all the Green's functions of the model can be obtained in closed form) and electric probe charges are screened due to the mass acquired by the gauge boson due to fermionic fluctuations [1–4]. On the other hand, in the quantum theory with massive fermions (which is not exactly solvable), electric probe charges interact via an effective potential that features both, a screening piece and a linear confining term. For large inter-charge separations the confining term prevails as long as the theta-vacuum angle is different from π and the probe charges are not integer multiples of the dynamical fermionic charges, in which case the confining term vanishes [2–4].

The quantized versions of both, the massless and massive Schwinger models, possess exact bosonic representations in $1+1$ dimensions, where the fermions are replaced by scalar bosons. This quantum fermion-boson map, called *bosonization*, has been obtained in the literature by using different techniques, as for instance, by computing the fermionic determinant, through loop and derivative expansions, etc (see [5] for a review on the subject).

In this Letter we present a new path for obtaining the bosonized versions of both, the massless and massive Schwinger models. This new construction is realized by considering a condensation of electric charges in $1+1$ dimensions via the Julia-Toulouse approach (JTA) for condensation of charges and defects [15, 16, 25, 26]. The massless Schwinger model is obtained when there are no vortices over the electric condensate (complete condensation) and the massive Schwinger model is obtained by taking into account the contribution of these defects (incomplete condensation).

The 1-form A_1 is the gauge connection with maximal rank that one can define in $1+1$ dimensions. As observed in [8], the general properties of the bosonized version of the Schwinger model are associated with this fact and, consequently, these general properties could be extended for arbitrary $d+1$ dimensions by working in terms of a maximal rank gauge connection A_d. By taking this observation into account, we show that the bosonized versions of the massless and massive Schwinger models can be straightforwardly generalized for arbitrary $d+1$ spacetime dimensions by considering the condensation of d-currents J_d minimally coupled to a maximal rank gauge connection A_d. Such a $(d+1)$-dimensional generalization of the bosonized versions of the massless and massive Schwinger models, however, contrary to what happens in $1+1$ dimensions, is not associated to a fermion-boson map in higher dimensional spacetimes. Indeed, as we shall discuss, the $(3+1)$-dimensional extension of the bosonized version of the massive Schwinger model corresponds to the large N chiral dynamics of the $(3+1)$-dimensional $SU(N)$ QCD in the limit $N\to\infty$, extending a previous observation reported in [8].

For uses of the maximal rank gauge connection in other physical scenarios, see for example [13].

The JTA [15, 16] is a prescription used to construct a low energy effective theory describing a system with condensed charges or defects, having previous knowledge of the model that describes the system in the regime with diluted charges or defects and also of the symmetries expected for the condensed regime. Based mainly on [15, 16], and taking also into account the ideas developed in [17, 18] regarding the formulation of ensembles of charges and defects, we introduced in [25, 26] a generalization of the JTA, which we shall use in this Letter. In particular, we are going to work with the dual JTA [25], which is defined in the dual picture to the one originally proposed in [16].

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II. THE BOSONIZED SCHWINGER MODEL AS AN ELECTRIC CONDENSATE

In this Letter we shall use natural units of $c = \hbar = 1$. We begin working in Minkowski spacetime $\mathbb{R}^{1,1}$.

The partition function describing the interaction of a gauge boson with external electric charges dilutely distributed through the space is given by:

$$Z_d[J_1] = \int_{G.F.} \mathcal{D}A_1 \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 + \right.\right. \\ \left.\left. -eA_1 \wedge *J_1 \right] \right\}, \tag{1}$$

where $J_1 = \delta\Sigma_2$ is the electric current that localizes the world-line of the electric charge e, the physical boundary of the world-surface of the electric Dirac string [22] (electric Dirac brane) localized by the Chern-Kernel Σ_2. The acronym "G.F." in the functional integral stands for some arbitrary gauge fixing procedure that must be implemented at some stage of the calculations.

By integrating out the gauge field we obtain a Coulomb interaction between the classical electric charges, which is confining in this dimensionality. This is easy to understand in physical terms: with only one spatial dimension, the electric field flows as it was in the interior of a straight flux tube and, consequently, the Coulomb potential is linear in the inter-charges separation in $1+1$ dimensions. However, this picture is changed when we take quantum effects into account. For this sake, we shall apply the JTA in the sequel to study the effects produced by a condensation of electric charges (after this step, one can include electric probe charges into the system and compare the new results with the classical confining picture).

To implement the JTA, we begin by adding to the Boltzmann factor in (1) an activation term for the electric currents (which effectively gives dynamics to the electric Dirac branes) such that it preserves the relevant symmetries of the system (P, T, Lorentz and the local gauge symmetry) and gives the dominant contribution for the dynamics of the electric condensate in the low energy regime [18, 25, 26]:

$$\begin{aligned} S_{activation}[J_1] &= \int_{\mathbb{R}^{1,1}} \frac{-1}{2\Lambda} J_1 \wedge *J_1 \\ &= \int_{\mathbb{R}^{1,1}} \frac{1}{2\Lambda} d*\Sigma_2 \wedge *d*\Sigma_2, \end{aligned} \tag{2}$$

where Λ is an adimensional free parameter of the JTA which we shall fix afterwards by comparison with the results obtained by using bosonization techniques. Introducing also a formal sum over (the branes Poincare-dual to) $*\Sigma_2$, we obtain the partition function defining the electric condensed regime:

$$Z_c := \sum_{\{*\Sigma_2\}} \int_{G.F.} \mathcal{D}A_1 \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 + \right.\right. \\ \left.\left. -eA_1 \wedge d*\Sigma_2 + \frac{1}{2\Lambda} d*\Sigma_2 \wedge *d*\Sigma_2 \right] \right\}. \tag{3}$$

Introducing into (3) the identity $\mathbb{1} = \int \mathcal{D}*P_2 \delta[*P_2 - *\Sigma_2]$, we rewrite the partition function for the electric condensed regime as:

$$Z_c = \int_{G.F.} \mathcal{D}A_1 \mathcal{D}*P_2 \left(\sum_{\{*\Sigma_2\}} \delta[*P_2 - *\Sigma_2] \right) \\ \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 - eA_1 \wedge d*P_2 + \right.\right. \\ \left.\left. + \frac{1}{2\Lambda} d*P_2 \wedge *d*P_2 \right] \right\}. \tag{4}$$

Next, we make use of a generalized version of the Poisson's identity (GPI) [18, 24]:

$$\begin{aligned} \sum_{\{*\Sigma_2\}} \delta[*P_2 - *\Sigma_2] &= \sum_{\{*\Omega_0\}} \exp\left\{ 2\pi i \int_{\mathbb{R}^{1,1}} *\Omega_0 \wedge *P_2 \right\} \\ &= \sum_{\{\Omega_0\}} \exp\left\{ 2\pi i \int_{\mathbb{R}^{1,1}} d^2x\, \Omega_0 (*P_2) \right\}, \end{aligned} \tag{5}$$

where Ω_0 is the brane Poisson-dual to Σ_2. The GPI works as a geometric analogue of the Fourier transform: when the brane configurations on the left hand side of (5) proliferate (condense), the brane configurations on the right hand side become diluted and vice-versa (see appendix A of [24] for a detailed discussion and derivation of the GPI in the general case). Hence, the proliferation of the electric Dirac branes Σ_2 (which is directly associated to the proliferation of the electric currents J_1 that live on their boundaries) is accompanied by the dilution of the branes of complementary dimension Ω_0 and vice-versa, what tells us that the branes Ω_0 must be interpreted as vortices (defects) over the electric condensate.

Using (5) and redefining $*P_2 =: \sqrt{\Lambda}\phi$, we rewrite (4) as:

$$Z_c = \int_{G.F.} \mathcal{D}A_1 \mathcal{D}\phi \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 + \right.\right. \\ \left.\left. -e\sqrt{\Lambda}\, A_1 \wedge d\phi + \frac{1}{2} d\phi \wedge *d\phi \right] \right\} Z_V[\phi], \tag{6}$$

where:

$$Z_V[\phi] = \sum_{\{\Omega_0\}} \exp\left\{ i \int_{\mathbb{R}^{1,1}} d^2x\, 2\pi\sqrt{\Lambda}\, \Omega_0 \phi \right\}, \tag{7}$$

is the vortex partition function. Equations (6) and (7) constitute the result of the use of the JTA for deriving the effective low energy theory of an electric condensate in $1+1$ dimensions, being ϕ the scalar field describing the electric condensate.

Now, to make explicit contact with the Schwinger model, we must consider the vortex contribution formally encoded in (7).

- **Case 1: The massless Schwinger model**

Let us suppose that the system is in a state with complete electric condensation, i.e., there are no vortices over the electric condensate such that the partition function (7) is trivial ($\Omega_0 \to 0 \Rightarrow Z_V[\phi] \to \mathbb{1}$). In this case, the partition function (6) gives the bosonized version of the massless Schwinger model:

$$Z_c^{m=0} = \int_{G.F.} \mathcal{D}A_1 \mathcal{D}\phi \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 + \right.\right.$$
$$\left.\left. - \frac{e}{\sqrt{\pi}} A_1 \wedge d\phi + \frac{1}{2} d\phi \wedge *d\phi \right] \right\}, \qquad (8)$$

where we fixed the JTA parameter $\Lambda = \pi^{-1}$ by comparison with the result obtained using bosonization techniques (see, for example, [3]).

By including electric probe charges into this system via a minimal coupling with the gauge field, one finds that the probe charges are screened by the electric condensate (see, for example, [4] for a detailed discussion). This result already shows that the classical confining picture is changed when quantum effects (electric condensation) are taken into account.

- **Case 2: The massive Schwinger model**

Let us suppose now that the system is in a state with incomplete electric condensation, i.e., there are vortices over the electric condensate such that the partition function (7) is not trivial. In the present case, one can give a precise prescription to realize the sum over point-vortices formally encoded in (7). For this sake, we are going to adopt the dilute gas approximation [6, 7].

We begin by Wick-rotating (7) to the euclidean space $\mathbb{R}^2$ ($t \mapsto -it_E, d^2x \mapsto -id^2x_E, \Omega_0 \mapsto -i\Omega_0^E, \phi \mapsto \phi_E$), where the sum over vortices is translated into a sum over instantons [6]:

$$Z_V^E[\phi_E] = \sum_{\{\Omega_0^E\}} \exp\left\{ -i \int_{\mathbb{R}^2} d^2x_E \, 2\sqrt{\pi}\, \Omega_0^E \phi_E \right\}, \qquad (9)$$

where we used that $\Lambda = \pi^{-1}$. First we consider the contribution of a single instanton with winding number $+1$ for the partition function $Z_V^E[\phi_E]$:

$$\int_{\mathbb{R}^2} d^2x_E^\alpha \, z \exp\left\{ -i \int_{\mathbb{R}^2} d^2x_E \, 2\sqrt{\pi}\, \delta^{(2)}(x_E - x_E^\alpha) \phi_E(x_E) \right\} =$$
$$= \int_{\mathbb{R}^2} d^2x_E^\alpha \, z \exp\left\{ -i2\sqrt{\pi}\, \phi_E(x_E^\alpha) \right\}, \qquad (10)$$

where z is the vortex fugacity (which gives the probability density of existence of a single instanton in space-time with winding number $+1$) and we are integrating over all the possible locations x_E^α of the instanton. Summing over all the possible configurations of the system with an arbitrary number of instantons with winding number $+1$, assuming that the instantons do not interact among themselves, we have for $Z_V^E[\phi_E]$:

$$\sum_{N_+=0}^{\infty} \frac{1}{N_+!} \left(\int_{\mathbb{R}^2} d^2x_E^\alpha \, z \exp\left\{ -i2\sqrt{\pi}\, \phi_E(x_E^\alpha) \right\} \right)^{N_+} =$$
$$= \exp\left\{ \int_{\mathbb{R}^2} d^2x_E \, z e^{-i2\sqrt{\pi}\, \phi_E(x_E)} \right\}, \qquad (11)$$

where the factor $(N_+!)^{-1}$ was introduced due to the fact that the instantons are indistinguishable. Taking also into account the contribution of an arbitrary number of antinstantons with winding number -1 and neglecting the contribution of instantons and antinstantons with higher winding numbers (which are exponentially supressed in the partition function if we assume a small fugacity), we get (see also [25]):

$$Z_V^E[\phi_E] \approx \exp\left\{ \int_{\mathbb{R}^2} d^2x_E \, z e^{-i2\sqrt{\pi}\, \phi_E(x_E)} \right\} \times$$
$$\times \exp\left\{ \int_{\mathbb{R}^2} d^2x_E \, z e^{+i2\sqrt{\pi}\, \phi_E(x_E)} \right\} =$$
$$= \exp\left\{ \int_{\mathbb{R}^2} d^2x_E \, 2z \cos(2\sqrt{\pi}\, \phi_E(x_E)) \right\}. \qquad (12)$$

Hence, the net effect of the instanton or vortex contribution is to generate a cossine for the scalar field [6, 7]. Wick-rotating (12) back to Minkowski and substituting the result into (6), we get the bosonized version of the massive Schwinger model:

$$Z_c^{m\neq 0} = \int_{G.F.} \mathcal{D}A_1 \mathcal{D}\phi \exp\left\{ i \int_{\mathbb{R}^{1,1}} \left[-\frac{1}{2} dA_1 \wedge *dA_1 + \right.\right.$$
$$\left.\left. - \frac{e}{\sqrt{\pi}} A_1 \wedge d\phi + \frac{1}{2} d\phi \wedge *d\phi + 2z\cos(2\sqrt{\pi}\,\phi) d^2x \right] \right\}. \qquad (13)$$

If we compare (13) with the result obtained using bosonization techniques, we fix the vortex fugacity to be $z = \frac{me\exp(\gamma)}{4\pi^{3/2}}$, where m is the fermion mass and γ is the Euler constant (see, for example, [3]). We then realize that the condition for small fugacity (which allows one to ignore the contribution of vortices with higher winding numbers) corresponds to the small coupling regime.

By including electric probe charges into this system via a minimal coupling with the gauge field, one finds that the probe charges interact via an approximate effective potential that features two parts: a screening piece plus a linear confining term. The confining term prevails for large inter-charge separations, as long as the theta-vacuum angle is different from π and the probe charges are not integer multiples of the charge of the electric condensate, in which case the confining term vanishes, restoring the screening phase. The theta-vaccum angle is introduced in the calculations as an integration constant in the evaluation of the interaction potential between the probe charges and corresponds to a (generally)

non-vanishing background electric field (see, for example, [4] for a detailed discussion).

The connection between the vortex contribution and the mass of the fermions in $1+1$ dimensions is detailed discussed in [6]. There, it is pointed out that an index theorem establishes the equality between the index of the massless Dirac operator (that is given by the difference between zero modes of the Dirac operator with positive and negative chiralities) and the Pontryagin index (topological charge or winding number) of the vortices. In the presence of vortices, this topological charge would be non-vanishing and, hence, there would be necessarily null eigenvalues of the massless Dirac operator in the fermionic determinant, in which case it would vanish. Therefore, for the massless Schwinger model, the vortex contribution is completely suppressed by the massless fermions. The situation is quite different in the massive case, since the massive Dirac operator has no zero modes and the vortex contribution is non-trivial in this case.

III. $(d+1)$-DIMENSIONAL GENERALIZATION OF THE BOSONIZED SCHWINGER MODEL VIA JTA

One of the great advantages of the JTA is that it allows a straightforward generalization of the preceding construction for arbitrary $d+1$ spacetime dimensions. For this sake, we begin by considering the partition function describing the interaction of a maximal rank gauge connection A_d with external topological currents J_d dilutely distributed through a $(d+1)$-dimensional Minkowski spacetime with metric diag$(-,+,\cdots,+)$ [27]:

$$Z_d[J_d] = \int_{G.F.} \mathcal{D}A_d \exp\left\{ i \int_{\mathbb{R}^{1,d}} \left[\frac{(-1)^d}{2} dA_d \wedge *dA_d + -eA_d \wedge *J_d \right] \right\}, \tag{14}$$

where $J_d = \delta\Sigma_{d+1}$. The JTA is implemented, as before, by adding to the Boltzmann factor in (14) the following activation term for the topological condensing d-currents:

$$\begin{aligned} S_{activation}[J_d] &= \int_{\mathbb{R}^{1,d}} \frac{-1}{2\lambda^{1-d}} J_d \wedge *J_d \\ &= \int_{\mathbb{R}^{1,d}} \frac{1}{2\lambda^{1-d}} d*\Sigma_{d+1} \wedge *d*\Sigma_{d+1}, \end{aligned} \tag{15}$$

where λ is a phenomenological JTA parameter with mass dimension, being the partition function for the condensed regime given by:

$$Z_c := \sum_{\{*\Sigma_{d+1}\}} \int_{G.F.} \mathcal{D}A_d \exp\left\{ i \int_{\mathbb{R}^{1,d}} \left[\frac{(-1)^d}{2} dA_d \wedge *dA_d + +(-1)^d eA_d \wedge d*\Sigma_{d+1} + \frac{1}{2\lambda^{1-d}} d*\Sigma_{d+1} \wedge *d*\Sigma_{d+1} \right] \right\}. \tag{16}$$

By repeating the same steps between equations (3) and (6), we rewrite (16) as:

$$Z_c = \int_{G.F.} \mathcal{D}A_d \mathcal{D}\phi \exp\left\{ i \int_{\mathbb{R}^{1,d}} \left[\frac{(-1)^d}{2} dA_d \wedge *dA_d + +(-1)^d mA_d \wedge d\phi + \frac{1}{2} d\phi \wedge *d\phi \right] \right\} Z_V[\phi], \tag{17}$$

where $m := e\lambda^{(1-d)/2}$ is the topological mass [28] generated by the condensation of topological d-currents and:

$$Z_V[\phi] = \sum_{\{\Omega_0\}} \exp\left\{ i \int_{\mathbb{R}^{1,d}} d^{d+1}x \, 2\pi\lambda^{(1-d)/2} \, \Omega_0 \phi \right\}, \tag{18}$$

is the vortex partition function. As before, we can evaluate the vortex contribution approximately by considering a small vortex fugacity z and the $(d+1)$-dimensional generalization of the bosonized version of the massive Schwinger model reads:

$$Z_c = \int_{G.F.} \mathcal{D}A_d \mathcal{D}\phi \exp\left\{ i \int_{\mathbb{R}^{1,d}} \left[\frac{(-1)^d}{2} dA_d \wedge *dA_d + + (-1)^d mA_d \wedge d\phi + \frac{1}{2} d\phi \wedge *d\phi + +2z\cos(2\pi\lambda^{(1-d)/2}\phi) d^{d+1}x \right] \right\}. \tag{19}$$

The $(d+1)$-dimensional generalization of the bosonized version of the massless Schwinger model, corresponding to a complete condensation of d-currents, is recovered from (19) by taking the vortex fugacity z to vanish.

IV. APPLICATION: THE $d=3$ CASE AND LARGE N CHIRAL DYNAMICS

Let us now consider the $d=3$ case of the preceding construction and its connection with the large N chiral dynamics of the $(3+1)$-dimensional $SU(N)$ QCD [11].

By writing down the vacuum-to-vacuum transition amplitude in a given theta-vacuum for QCD, one identifies a CP violating term given by [9]:

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} \mathrm{Tr}[\mathbb{F}_2 \wedge \mathbb{F}_2], \tag{20}$$

where g is the QCD coupling constant, $\mathbb{F}_2 = d\mathbb{A}_1 + \mathbb{A}_1 \wedge \mathbb{A}_1$ and $\mathbb{A}_1 = A_1^a \mathbb{T}^a$, with $\{\mathbb{T}^a\}$ being the set of the N^2-1 generators of the $\mathfrak{su}(N)$ algebra. The action associated to the Lagrangian density (20) is a surface term, which is non-vanishing due to instanton configurations of the non-Abelian connection.

It was pointed out in [10] that one can rewrite the theta-term (20) in terms of an Abelian 3-form A_3 according to:

$$\mathcal{L}_\theta = \theta dA_3 = -\theta * F, \tag{21}$$

where $F := *F_4 = *dA_3$, being the Abelian 3-form A_3 a composite field defined by the trace of the non-Abelian Chern-Simons 3-form. It can be shown that under an arbitrary $SU(N)$ gauge transformation, A_3 transforms like an Abelian connection; furthermore, it can be shown that the 2-point correlation function of the composite field A_3 corresponds to a Coulomb propagator, and hence A_3 behaves as a massless colorless collective field propagating a long-range interaction [10, 12]. At this point, one could aim to construct an effective action for the Abelian field A_3. The simplest one corresponds to the very low energy limit (where the masses of the quarks and hence, the masses of the mesons, are taken to infinity) of the very large N approximation for the effective action of QCD describing the pseudo-Goldstone bosons (pseudoscalar mesons) associated to the chiral symmetry breaking produced by the quark condensate [12]:

$$S_d^{eff}[A_3, J_3] \sim \int_{\mathbb{R}^{1,3}} \left[-\frac{1}{2} dA_3 \wedge *dA_3 + \theta dA_3 - eA_3 \wedge *J_3\right], \tag{22}$$

where the mesons were integrated out in the above referred approximations and we also added the last term in (22) corresponding to a source J_3 for the Abelian field A_3. But this, apart from the theta-term, is exactly the diluted phase for the sources J_3 in the $d = 3$ case (14).

It is important to notice that the field strength $F = *F_4 = *dA_3$ propagates no dynamical degrees of freedom in $3+1$ dimensions, since it can be shown from the equations of motion coming from (22), that F is just a constant in spacetime [12]. Hence, the composite field A_3, although responsible for setting a constant background field F into the theory, does not imply in the presence of any dynamical massless colorless collective excitations of the gluons in the QCD spectrum, what is desirable, otherwise it would enter in conflict with the expectation that there is a mass gap in QCD [12].

If the sources J_3 undergo an incomplete condensation process, there being vortices (instantons) over the bubble condensate, the effective action for the condensed phase can be read off from (19), taking into account the theta-term coming from (22):

$$S_c^{eff}[A_3, \phi] = \int_{\mathbb{R}^{1,3}} \left[-\frac{1}{2} dA_3 \wedge *dA_3 + \theta dA_3 - mA_3 \wedge d\phi + \right.$$
$$\left. + \frac{1}{2} d\phi \wedge *d\phi + 2z \cos\left(\frac{2\pi}{\lambda}\phi\right) d^4x\right]. \tag{23}$$

Integrating out A_3, we obtain:

$$S_c^{eff}[\phi] = \int_{\mathbb{R}^{1,3}} \left[\frac{1}{2} d\phi \wedge *d\phi - V(\phi) d^4x\right], \tag{24}$$

where the potential energy is given by:

$$V(\phi) = \frac{1}{2}(m\phi - \theta)^2 - 2z \cos\left(\frac{2\pi}{\lambda}\phi\right). \tag{25}$$

Redefining $\phi \mapsto \frac{\lambda}{2\pi}\phi$ (which gives a trivial Jacobian in the path integral) and absorbing a constant factor of $\frac{2\pi}{e}$ into the arbitrary constant θ-parameter, we rewrite (25) as:

$$V(\phi) = \frac{e^2}{8\pi^2}(\phi - \theta)^2 - 2z \cos(\phi), \tag{26}$$

which has exactly the same form of the potential energy obtained in [11] when considering the large N effective action for QCD with the quark masses fixed and the limit $N \to \infty$, case in which the different flavors are decoupled and the potential energy (26) refers to a given flavor. In [11], the pseudoscalar field ϕ describes a single meson (pseudo-Goldstone boson of the chiral symmetry breaking) of the one flavor potential (26), with $\frac{e^2}{8\pi^2} = \frac{aF_\pi^2}{2N}$ and $2z = F_\pi^2 \mu^2$, where F_π is the meson decay constant, μ is a constant contributing to the meson mass and a is a constant of order 1 when $N \to \infty$.

As observed in [11], the potential energy (26) is the same one obtained by bosonizing the massive Schwinger model in $d = 1$, a fact that is made clear in the formalism developed here, given that the effective action (23) is the generalization of the bosonized version of the massive Schwinger model generated via the JTA (19) for the $d = 3$ case. Notice also that the generalized version of the massless Schwinger model in $d = 3$ was already reported in [8] to be connected with the large N chiral dynamics of $SU(N)$ QCD. As commented below, this is the case for a massless quark. Here we extended this connection by considering the generalized version of the massive Schwinger model in $d = 3$, which corresponds to the realistic case with massive quarks.

Let us put some results into perspective. As stated before, the effective action (22) is the limit of the effective action (23) when the meson mass is very large. From the JTA point of view, given the above identifications of parameters, this means that the topological mass m in (23) is much lower than the vortex fugacity z, and hence there are many vortices (instantons) over the bubble condensate, which is effectively destroyed at large distances (low energy regime), making the system return to the diluted phase (22). Another interesting limit is seen when the meson mass goes to zero, which is equivalent to consider a massless quark. From the JTA point of view, this means that the vortex fugacity vanishes, and hence there are no instantons in the system (complete bubble condensation). As discussed in [11], when there is a massless quark in the system, the θ-parameter can be eliminated from the theory via field redefinitions and, in this case, there would be no strong CP violation: this is in consonance with the fact that in the JTA picture a massless quark would imply in the complete suppression of the instantons, which are the basic reason behind the strong CP violation.

V. CONCLUSION

In this Letter we showed how the bosonized versions of the massless and massive Schwinger models can be constructed via JTA by considering a condensation of electric charges in $1+1$ dimensions. The massless case is obtained when there are no vortices over the electric condensate (complete condensation) and the massive case is derived when these defects are present (incomplete condensation). We then discussed the $(d+1)$-dimensional generalization of both, the massless and massive bosonized Schwinger models, associating their emergence with the condensation of topological d-currents minimally coupled to a maximal rank gauge connection A_d. In $d=3$, this generalization gives the large N approximation for the effective action of $SU(N)$ QCD in the limit $N \to \infty$ [11].

As a final remark, we point out the fact that the electric condensate interpretation obtained here via JTA for fermionic radiative corrections in the 2-dimensional electrodynamics is analogous to the induction of the Chern-Simons term via JTA due to a P and T violating electric condensate in 3-dimensional electrodynamics [23, 25]: the Maxwell-Chern-Simons theory [19–21] constitutes the low energy effective theory derived by integrating out the fermionic degrees of freedom in $2+1$ dimensions. A mass term for the fermions in $2+1$ dimensions violates P and T and even for massless fermions these discrete spacetime symmetries are violated due to the parity anomaly. This is the reason for electric condensates with different symmetries in $1+1$ and $2+1$ dimensions. These two explicit examples of fermionic radiative corrections being described as electric condensation processes show that the JTA can be made more general than just a dual description of the Higgs mechanism as originally proposed in [16]. This conclusion is rather reinforced by the fact that there is no Higgs mechanism in $1+1$ dimensions [14], but still the Schwinger mechanism can be described via JTA as showed in this Letter. Also, the Higgs mechanism is a mass generation mechanism for 1-form gauge fields, while the mass generation mechanism associated to the condensation of arbitrary extended topological p-currents described by the JTA can give mass for arbitrary p-form gauge fields.

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